

# Minimax estimation: a non-asymptotic outlook

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Various non-asymptotic lower bounds in the minimax estimation of normal means, due to Van Trees, Chentsov, Bhattacharyya, Koores, Ibragimov-Khasminkii, Donoho, Casella-Strawderman, and others, will be reviewed and compared. A lower bound for the minimax risk will be proposed which, in a sense, combines all these inequalities. The bound will be shown to be highly efficient for any sample size.

These results will be applied next to continuous time nonparametric regression in the white Gaussian noise. Some general ellipsoidal and cuboidal functional classes will be introduced using Fourier transforms. These will include classes of entire functions of exponential type, Paley-Wiener classes, Sobolev classes, and their modifications. For a variety of such classes, the best linear estimators will be discussed and their efficiency will be numerically evaluated, in a non-asymptotic setting.

These two basic statistical models help to better understand role of the asymptotic approach in the modern age of computers. An offhand asymptotic approach to optimal estimation is shown to be often unreliable and, at times, misleading, in the case of moderate sample sizes. Some implications for adaptive nonparametric estimation of linear functionals will also be mentioned.